Module 3: Lecture 3d.

Maths of Social Welfare Functions

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Economics of Health and Health Care

Overview

Objective: Understand the maths underpinning social welfare functions

- Social Welfare Functions
- Social Indifference Curves
- Optimal Policy Choice
  - Maximise Social Welfare
  - Marginal Social Benefits

Social Welfare Functions

- Social welfare functions rank alternative distributions of health
- This can be used to make choices between health policies
- But what is a social welfare function?

Social Welfare Function

$$SW = \sum_{i=1}^{N} \omega_i U(h_i) \tag{1}$$

where:

$Individual : i \qquad Population\,Size : N \qquad Health : h_i$

$Weight : \omega_i \qquad Social\ Utility\ Function : U(.)$

Social Welfare Function - Population Size

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

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$N = 1$

$$SW = \omega_1 U(h_1)$$

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$N = 2$

$$SW = \omega_1 U(h_1) + \omega_2 U(h_2)$$

Social Welfare Function - Population Size

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$$SW = \omega_1 U(h_1) + \omega_2 U(h_2)$$

$N = 3$

$$SW = \omega_1 U(h_1) + \omega_2 U(h_2) + \omega_3 U(h_3)$$

Social Welfare Function - Weights

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Anonymous:

$$\omega_i = \frac{1}{N}$$

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Rank, $R(h_i)$:

$$\omega_i = \frac{2N - 2R(h_i) + 1}{N^2}$$

Social Welfare Function - Weights

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Anonymous:

$$\omega_i = \frac{1}{N}$$

Rank, $R(h_i)$:

$$\omega_i = \frac{2N - 2R(h_i) + 1}{N^2}$$

Socioeconomic-Rank, $R(SES_i)$:

$$\omega_i = \frac{2N - 2R(SES_i) + 1}{N^2}$$

Social Welfare Function - Social Utility Function

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

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Linear:

$$U(h_i) = h_i$$

Social Welfare Function - Social Utility Function

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Linear:

$$U(h_i) = h_i$$

Constant Relative Inequality-Aversion, ε:

$$U(h_i) = A + \frac{h_i^{1-\varepsilon}}{B(1-\varepsilon)}$$

Social Welfare Functions - Functional Forms

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

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$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

$$SW_{Atkinson} = \sum_{i=1}^{N} \frac{1}{N}\left(A + \frac{h_i^{1-\varepsilon}}{B(1-\varepsilon)}\right) \quad (2)$$

Social Welfare Functions - Functional Forms

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$$SW_{Rank} = \sum_{i=1}^{N} \frac{2N - 2R(h_i) + 1}{N^2} h_i \quad (3)$$

Social Welfare Functions - Functional Forms

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

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$$SW_{Rank} = \sum_{i=1}^{N} \frac{2N - 2R(h_i) + 1}{N^2} h_i \quad (3)$$

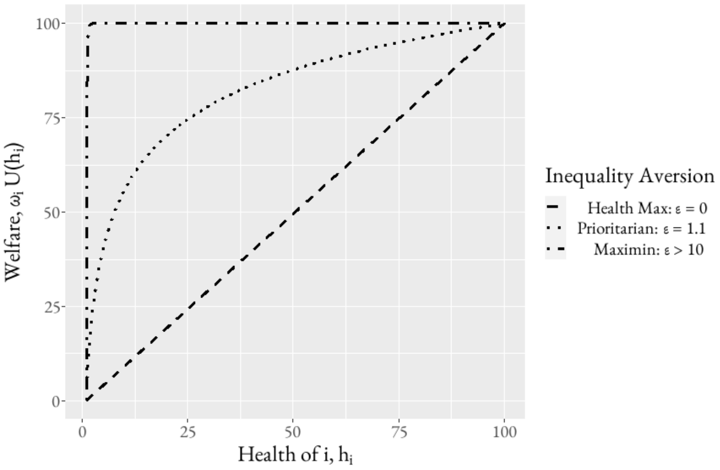
$$SW_{SESRank} = \sum_{i=1}^{N} \frac{2N - 2R(SES_i) + 1}{N^2} h_i \quad (4)$$

Social Welfare - Atkinson

$N = 1$

$$SW_{Atkinson} = \frac{1}{N}\left(A + \frac{h_1^{1-\varepsilon}}{B(1-\varepsilon)}\right)$$

Social Welfare

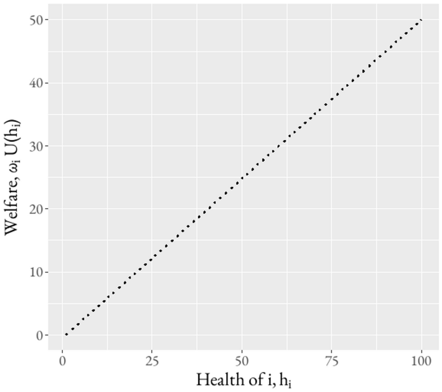


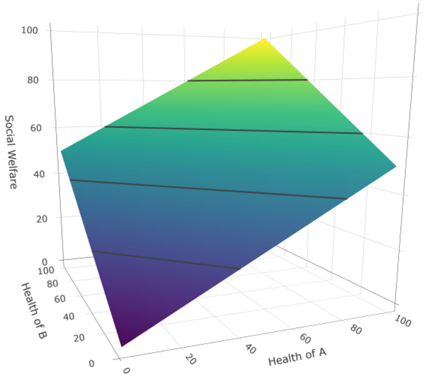
Social Welfare - Atkinson: Health Maximiser

$N = 2$, $\varepsilon = 0$

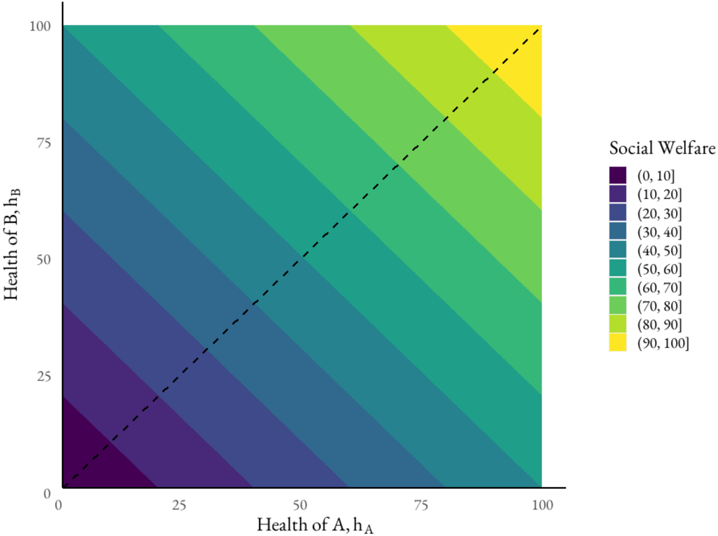
$$SW_{Atkinson} = \frac{1}{N}\left(A + \frac{h_1^{1-\varepsilon}}{B(1-\varepsilon)}\right) + \frac{1}{N}\left(A + \frac{h_2^{1-\varepsilon}}{B(1-\varepsilon)}\right)$$
$$= \frac{1}{2}\left(A + \frac{h_1}{B}\right) + \frac{1}{2}\left(A + \frac{h_2}{B}\right)$$

Social Welfare - Atkinson: Health Maximiser





Social Indifference Curves - Health Maximiser

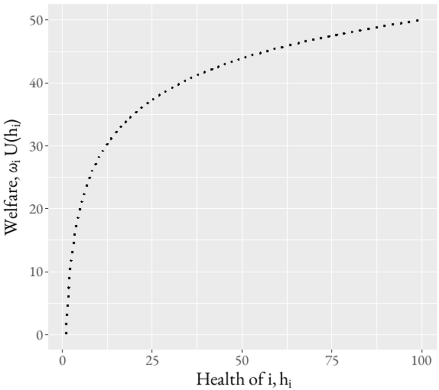


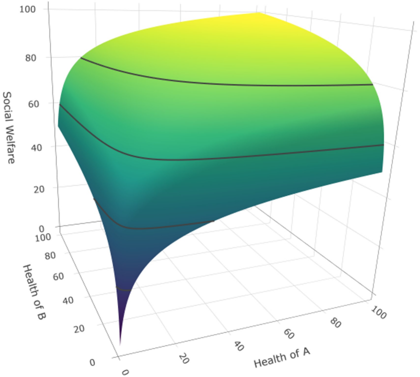
Social Welfare - Atkinson: Prioritarian

$N = 2$, $\varepsilon = 1.1$

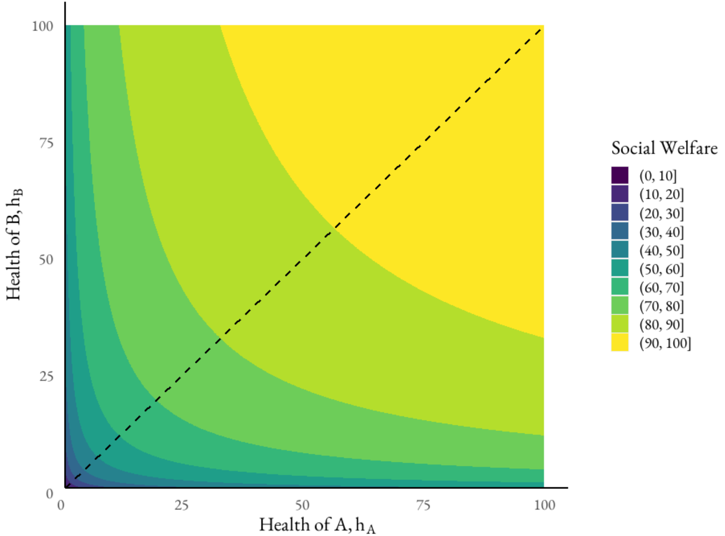
$$SW_{Atkinson} = \frac{1}{N}\left(A + \frac{h_1^{1-\varepsilon}}{B(1-\varepsilon)}\right) + \frac{1}{N}\left(A + \frac{h_2^{1-\varepsilon}}{B(1-\varepsilon)}\right)$$
$$= \frac{1}{2}\left(A + \frac{h_1^{-0.1}}{B(-0.1)}\right) + \frac{1}{2}\left(A + \frac{h_2^{-0.1}}{B(-0.1)}\right)$$

Social Welfare - Atkinson: Prioritarian





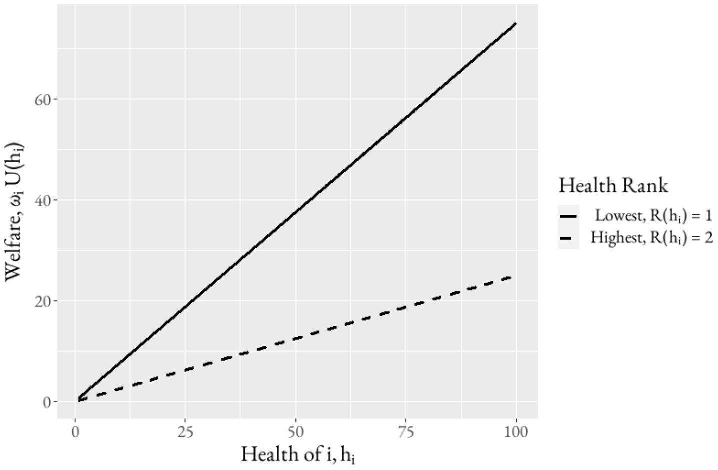
Social Indifference Curves - Prioritarian



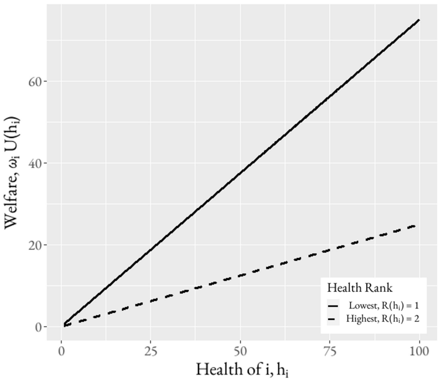
Social Welfare - Rank-Dependent

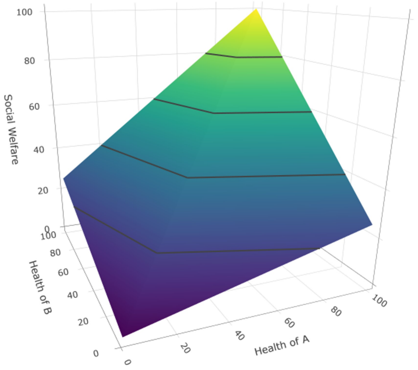
$$SW_{Rank} = \frac{2N - 2R(h_1) + 1}{N^2} h_1 + \frac{2N - 2R(h_2) + 1}{N^2} h_2$$

Social Welfare - Rank-Dependent

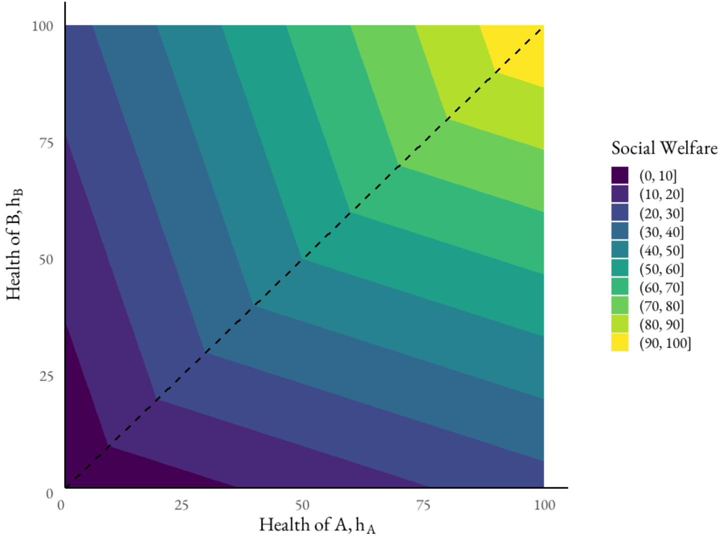


Social Welfare - Rank-Dependent





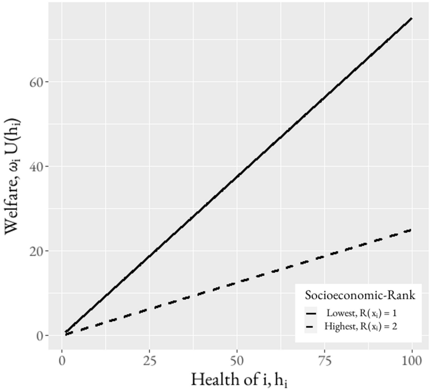
Social Indifference Curves - Rank-Dependent

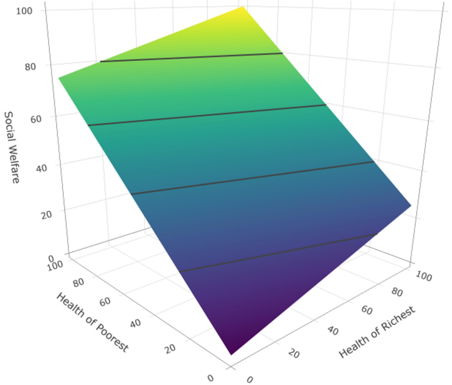


Social Welfare - Socioeconomic Rank-Dependent

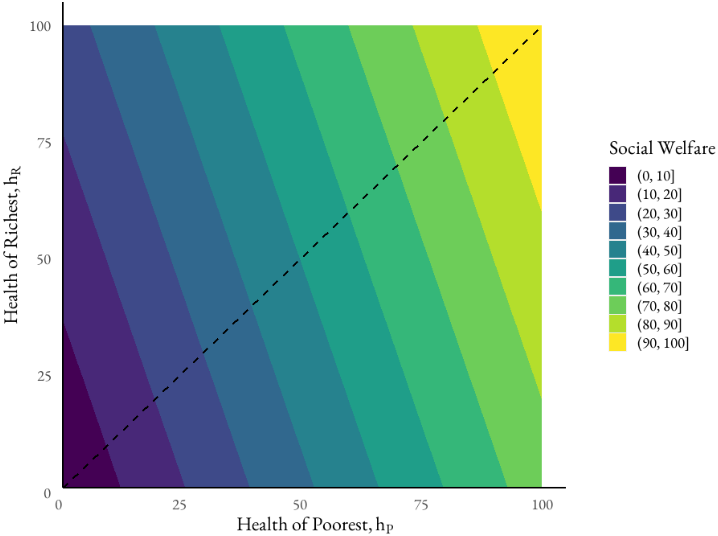
$$SW_{SESRank} = \frac{2N - 2R(SES_1) + 1}{N^2} h_1 + \frac{2N - 2R(SES_2) + 1}{N^2} h_2$$

Social Welfare - Rank-SES Dependent

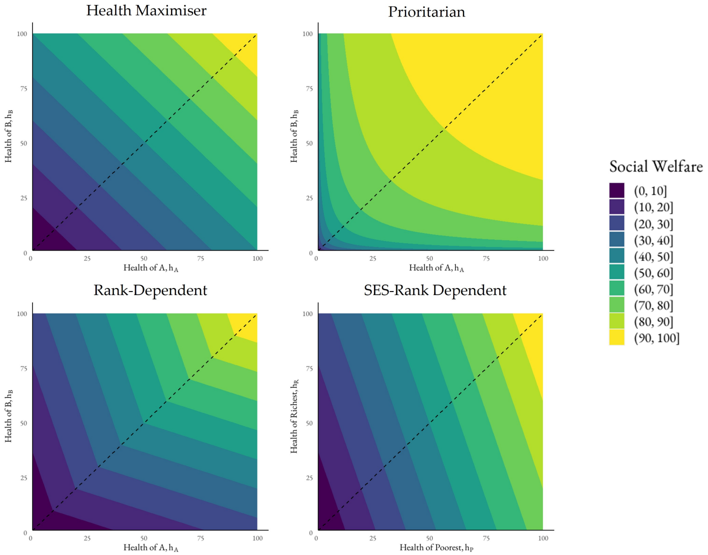




Social Indifference Curves - SES Rank-Dependent



Social Indifference Curves



Optimal Policy Choice

Two questions:

- Which policy is preferred?
- Whose health should be improved?

Optimal Policy Choice

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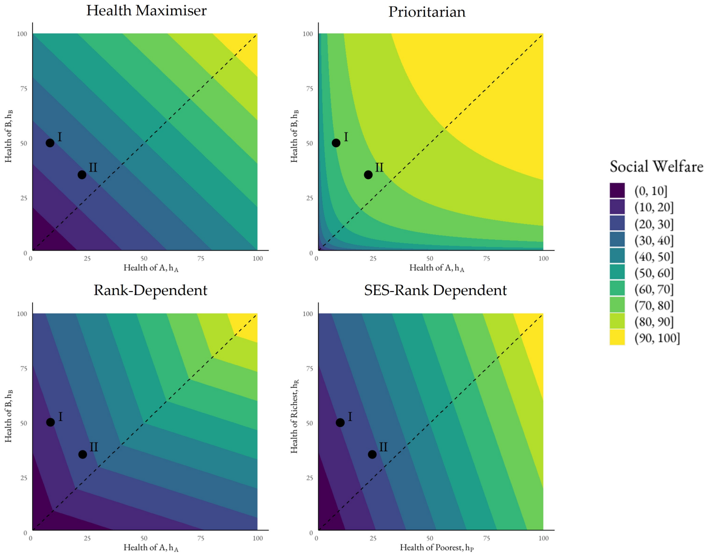
- Which policy is preferred?
- Whose health should be improved?

Answer: Maximise social welfare.

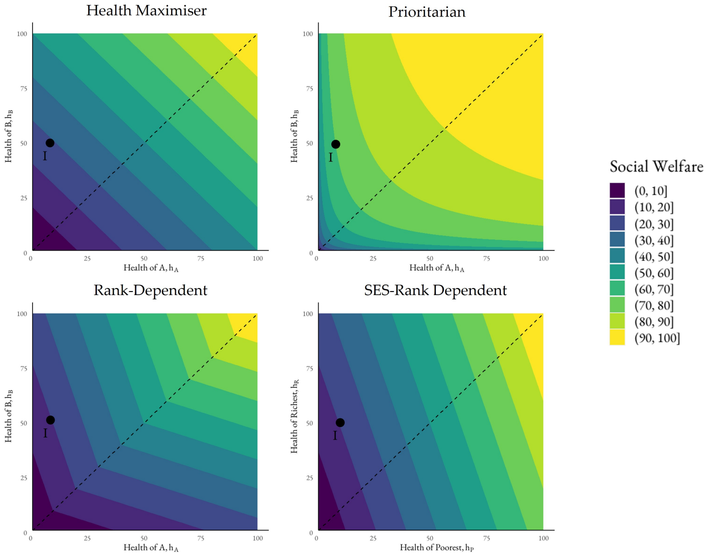
Preferred Policy?



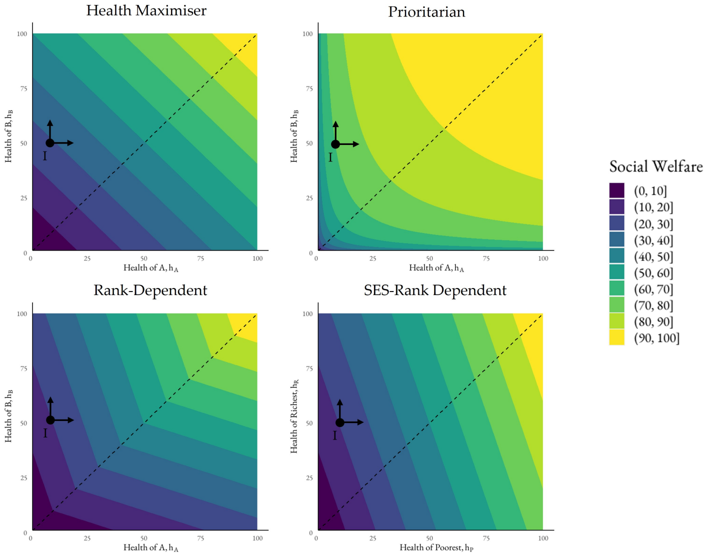
Preferred Policy?

	Social Welfare	
	I	II
Health Maximiser	*29.3*	*29.3*
Prioritarian	71.7	*77.5*
Rank-Dependent	20	*26.5*
SES Rank-Dependent	20	*26.5*

Whose health to improve?



Whose health to improve?



Whose health to improve?

Marginal Social Benefit: $MSB_i = \frac{\partial SW}{\partial h_i}$

- Increase in social welfare from a marginal improvement in health to i
- Improve health where the marginal benefit is highest

Marginal Social Benefit - Atkinson

$$SW_{Atkinson} = \frac{1}{N}\left(A + \frac{h_1^{1-\varepsilon}}{B(1-\varepsilon)}\right) + \frac{1}{N}\left(A + \frac{h_2^{1-\varepsilon}}{B(1-\varepsilon)}\right)$$

$$\frac{\partial SW_{Atkinson}}{\partial h_1} = \frac{1}{N}\frac{h_1^{-\varepsilon}}{B}$$

$$\frac{\partial SW_{Atkinson}}{\partial h_2} = \frac{1}{N}\frac{h_2^{-\varepsilon}}{B}$$

Marginal Social Benefit - Rank-Dependent

$$SW_{Rank} = \frac{2N - 2R(h_1) + 1}{N^2} h_1 + \frac{2N - 2R(h_2) + 1}{N^2} h_2$$

$$\frac{\partial SW_{Rank}}{\partial h_i} = \frac{2N - 2R(h_i) + 1}{N^2}$$

$$= \frac{3}{4} \quad if\ R(h_i) = 1$$

$$= \frac{1}{4} \quad if\ R(h_i) = 2$$

Marginal Social Benefit - Socieconomic Rank-Dependent

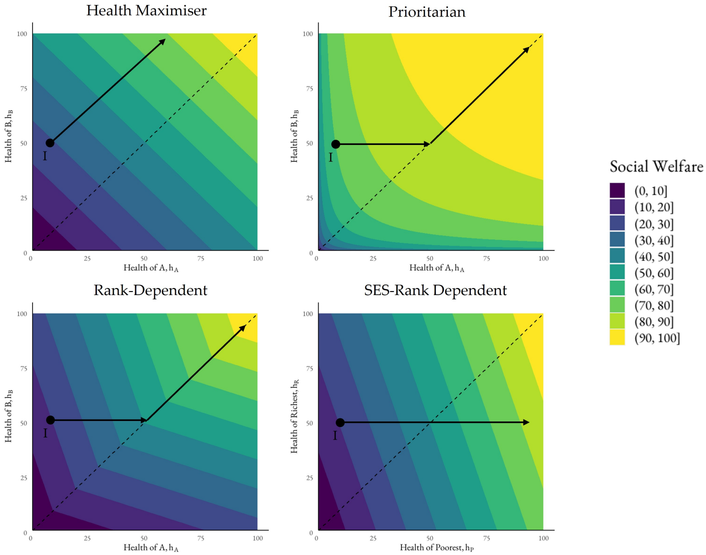
$$SW_{SESRank} = \frac{2N - 2R(SES_1) + 1}{N^2} h_1 + \frac{2N - 2R(SES_2) + 1}{N^2} h_2$$

$$\frac{\partial SW_{SESRank}}{\partial h_i} = \frac{2N - 2R(SES_i) + 1}{N^2}$$
$$= \frac{3}{4} \quad if\ R(SES_i) = 1$$
$$= \frac{1}{4} \quad if\ R(SES_i) = 2$$

Whose health to improve?

	Marginal Benefit	
	$h_A = 10$	$h_B = 50$
Health Maximiser	*0.5*	*0.5*
Prioritarian	*1.07*	0.18
Rank-Dependent	*0.75*	0.25
SES Rank-Dependent	*0.75*	0.25

Whose health to improve?



Overview

- Social welfare functions use weights and social utility functions to rank alternative health distributions
- Social indifference curves allow us to visualise social welfare across possible health distributions
- Optimal policy choice depends on maximising social welfare or marginal social benefits

References and Extended Reading

- Atkinson, A. B., (1970), On the measurement of inequality. *Journal of Economic Theory*, 2(3), 244-263.
- Wagstaff, A., (2002), Inequality aversion, health inequalities and health achievement. *Journal of Health Economics*, 21(4), 627-641.

Equally Distributed Equivalent

- The Equally Distributed Equivalent mentioned in the previous lecture is closely related to the Atkinson social welfare function shown in this lecture
- Both provide identical rankings of alternative distributions, but the EDE provides an intuitive level of welfare
- The next slide shows the relationship

Equally Distributed Equivalent

Given the social utility function:

$$U(h_i) = A + \frac{h_i^{1-\varepsilon}}{B(1-\varepsilon)} \tag{5}$$

And the social welfare function:

$$SW_{Atkinson} = \sum_{i=1}^{N} \frac{1}{N} \left(A + \frac{h_i^{1-\varepsilon}}{B(1-\varepsilon)} \right) \tag{6}$$

We apply the *inverse* of the social utility function to the social welfare function to get the equally distributed equivalent:

$$EDE_{Atkinson} = \left(\sum_{i=1}^{N} \frac{1}{N} h_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{7}$$

Whose health to improve?

Marginal Rate of Social Substitution: $MRSS_{ij} = \frac{\partial SW}{\partial h_i} / \frac{\partial SW}{\partial h_j}$

- Relative improvement in social welfare from a marginal improvement in health to i compared to a marginal improvement in health j
- Improve health of i if $MRSS_{ij} > 1$

Marginal Rate of Social Substitution - Atkinson

$$
\begin{aligned}
MRSS_{12} &= \frac{\partial SW_{Atkinson}}{\partial h_1} \Big/ \frac{\partial SW_{Atkinson}}{\partial h_2} \\
&= \frac{N}{N} \frac{h_1^{-\varepsilon}}{h_2^{-\varepsilon}} \\
&= \left(\frac{h_2}{h_1} \right)^{\varepsilon}
\end{aligned}
$$

Marginal Rate of Social Substitution - Rank-Dependent

$$
\begin{aligned}
MRSS_{12} &= \tfrac{\partial SW_{Rank}}{\partial h_1} \Big/ \tfrac{\partial SW_{Rank}}{\partial h_2} \\
&= \frac{2N - 2R(h_1) + 1}{2N - 2R(h_2) + 1} \\
&= 3 \quad if\ R(h_1) = 1\,,\ R(h_2) = 2 \\
&= \frac{1}{3} \quad if\ R(h_1) = 2\,,\ R(h_2) = 1
\end{aligned}
$$

Marginal Rate of Social Substitution - Socieconomic Rank-Dependent

$$
\begin{aligned}
MRSS_{12} &= \frac{\partial SW_{SESRank}}{\partial h_1} \Big/ \frac{\partial SW_{SESRank}}{\partial h_2} \\
&= \frac{2N - 2R(SES_1) + 1}{2N - 2R(SES_2) + 1} \\
&= 3 \quad if\ R(SES_1) = 1\,,\ R(SES_2) = 2 \\
&= \frac{1}{3} \quad if\ R(SES_1) = 2\,,\ R(SES_2) = 1
\end{aligned}
$$