Social Welfare Functions

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Overview

Objective: Understand what social welfare functions are

- Social Welfare Functions
- Equally Distributed Equivalent
- Policy Choice
- Social Indifference Curves
- Alternatives
- Policy Choice II

Social Welfare Functions

- Social welfare functions rank alternative distributions of health
- This can be used to make choices between health policies
- But what is a social welfare function?

Social Welfare Function

$$SW = \sum_{i=1}^{N} \omega_i U(h_i) \tag{1}$$

where:

$Individual : i$ $Population\ Size : N$ $Health : h_i$

$Weight : \omega_i$ $Social\ Utility\ Function : U(.)$

Social Welfare Function - Population Size

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

Social Welfare Function - Population Size

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$N = 1$

$$SW = \omega_1 U(h_1)$$

Social Welfare Function - Population Size

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$N = 2$

$$SW = \omega_1 U(h_1) + \omega_2 U(h_2)$$

Social Welfare Function - Population Size

$$SW = \sum_{i=1}^{N} \omega_i U(h_i)$$

$N = 1$

$$SW = \omega_1 U(h_1)$$

$N = 2$

$$SW = \omega_1 U(h_1) + \omega_2 U(h_2)$$

$N = 3$

$$SW = \omega_1 U(h_1) + \omega_2 U(h_2) + \omega_3 U(h_3)$$

Atkinson Social Welfare Function

$$SW = \sum_{i=1}^{N} \frac{1}{N} \frac{h_i^{1-\varepsilon} - 1}{1 - \varepsilon} \tag{2}$$

where:

$$Weight : \omega_i = \frac{1}{N}$$

$$Social\ Utility\ Function : U(h_i) = \frac{h_i^{1-\varepsilon} - 1}{1 - \varepsilon}$$

Health Inequality Aversion, ε

$$SW = \sum_{i=1}^{N} \frac{1}{N} \frac{h_i^{1-\varepsilon} - 1}{1 - \varepsilon} \tag{3}$$

$$\textit{Health Maximiser} : \varepsilon = 0$$

$$\textit{Prioritarian} : 0 < \varepsilon < \infty$$

$$\textit{Maximin} : \varepsilon = \infty$$

Equally Distributed Equivalent

The level of health is the level of health which, if equally distributed across all individuals in the population, would provide the same level of social welfare as the existing distribution of health.

Equally Distributed Equivalent

$$SW = \sum_{i=1}^{N} \frac{1}{N} \frac{h_i^{1-\varepsilon} - 1}{1 - \varepsilon} \tag{4}$$

$$\sum_{i=1}^{N} \frac{1}{N} \frac{h_{EDE}^{1-\varepsilon} - 1}{1 - \varepsilon} = \sum_{i=1}^{N} \frac{1}{N} \frac{h_i^{1-\varepsilon} - 1}{1 - \varepsilon} \tag{5}$$

Equally Distributed Equivalent

$$h_{EDE} = \left(\sum_{i=1}^{N} \frac{1}{N} h_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{6}$$

Health Inequality Aversion, ε

$$h_{EDE} = \left(\sum_{i=1}^{N} \frac{1}{N} h_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{7}$$

$$\textit{Health Maximiser} : \varepsilon = 0$$

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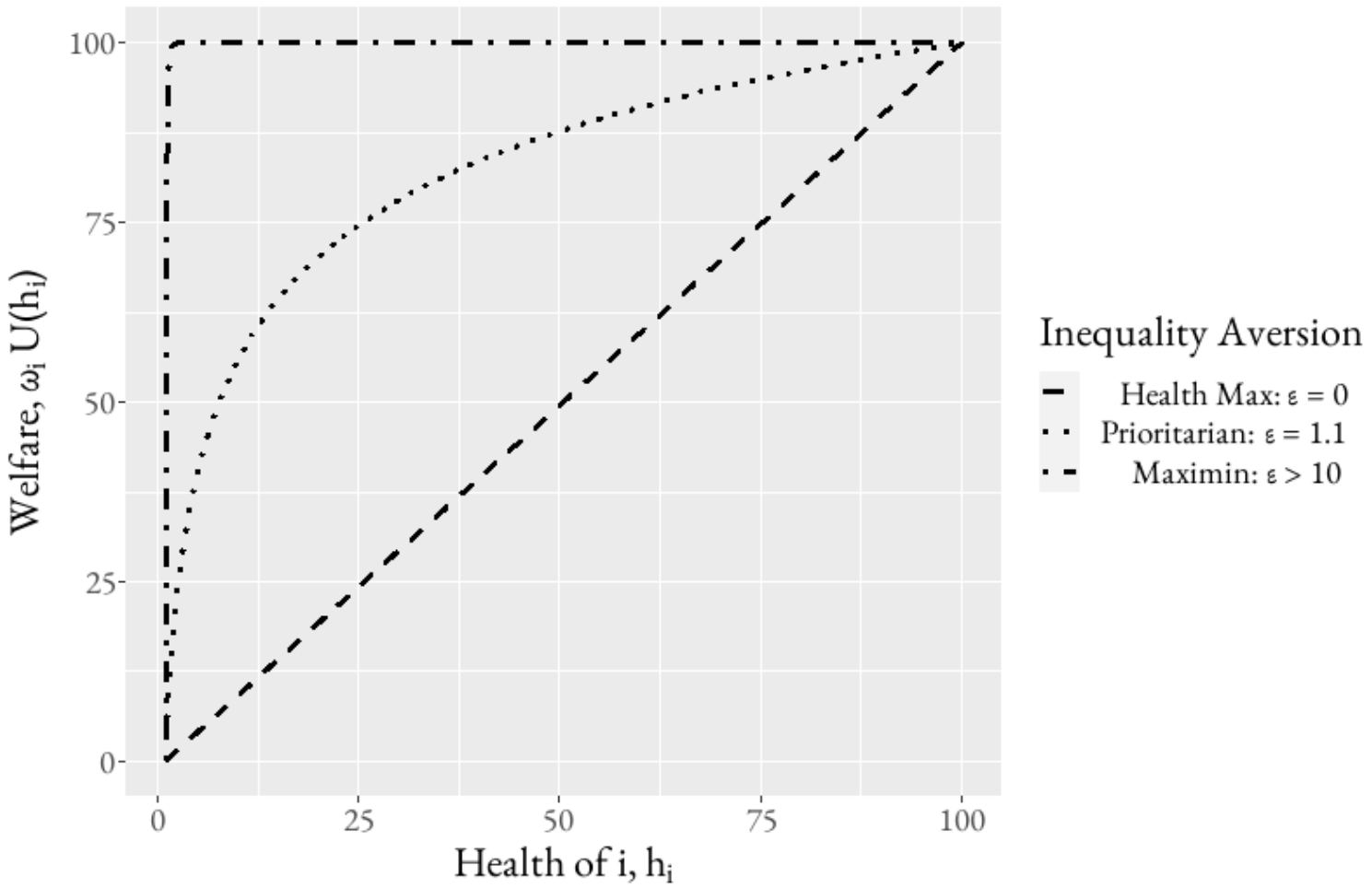
Table: Alternative Policies

Policy	**Health**	
	h_1	h_2
I	60	80
II	62	80
III	60	84

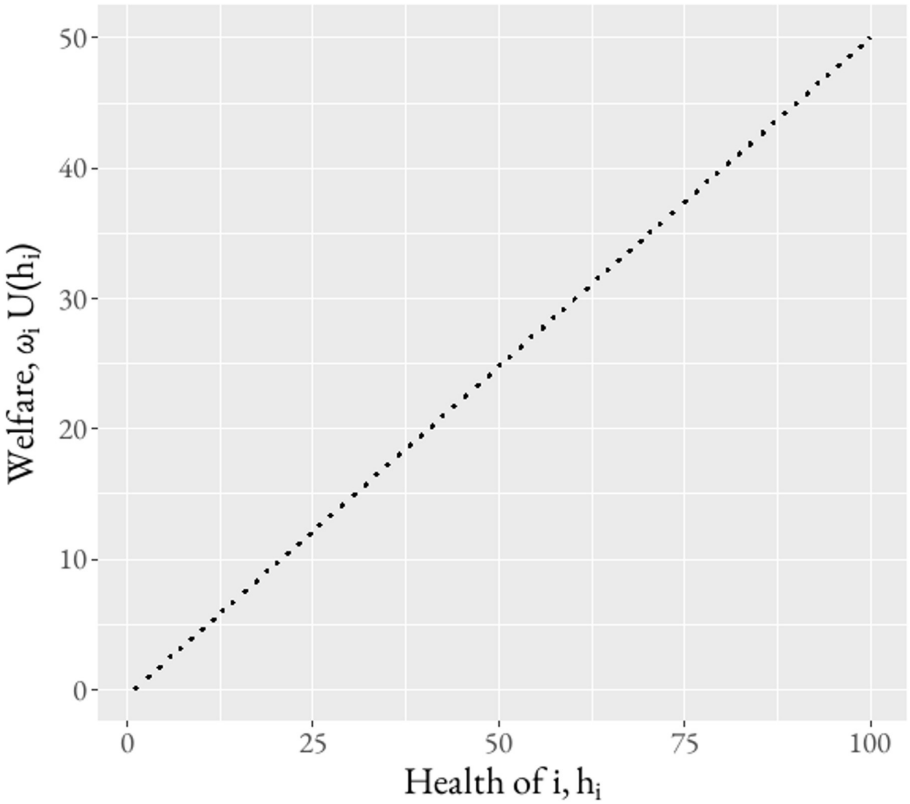
Table: Social Welfare

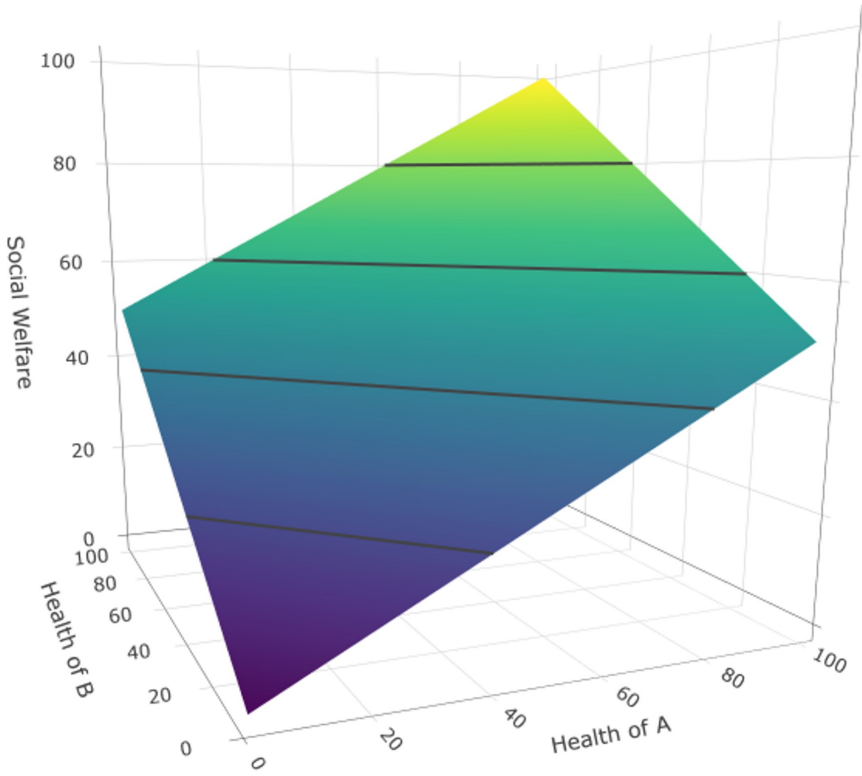
Policy	**Social Welfare, $h_{\mathbf{EDE}}$**		
	$\varepsilon = 0$	$\varepsilon = 2$	$\varepsilon = 100$
I	70.00	68.57	60.42
II	71.00	69.86	**62.44**
III	**72.00**	**70.00**	60.42

Social Welfare

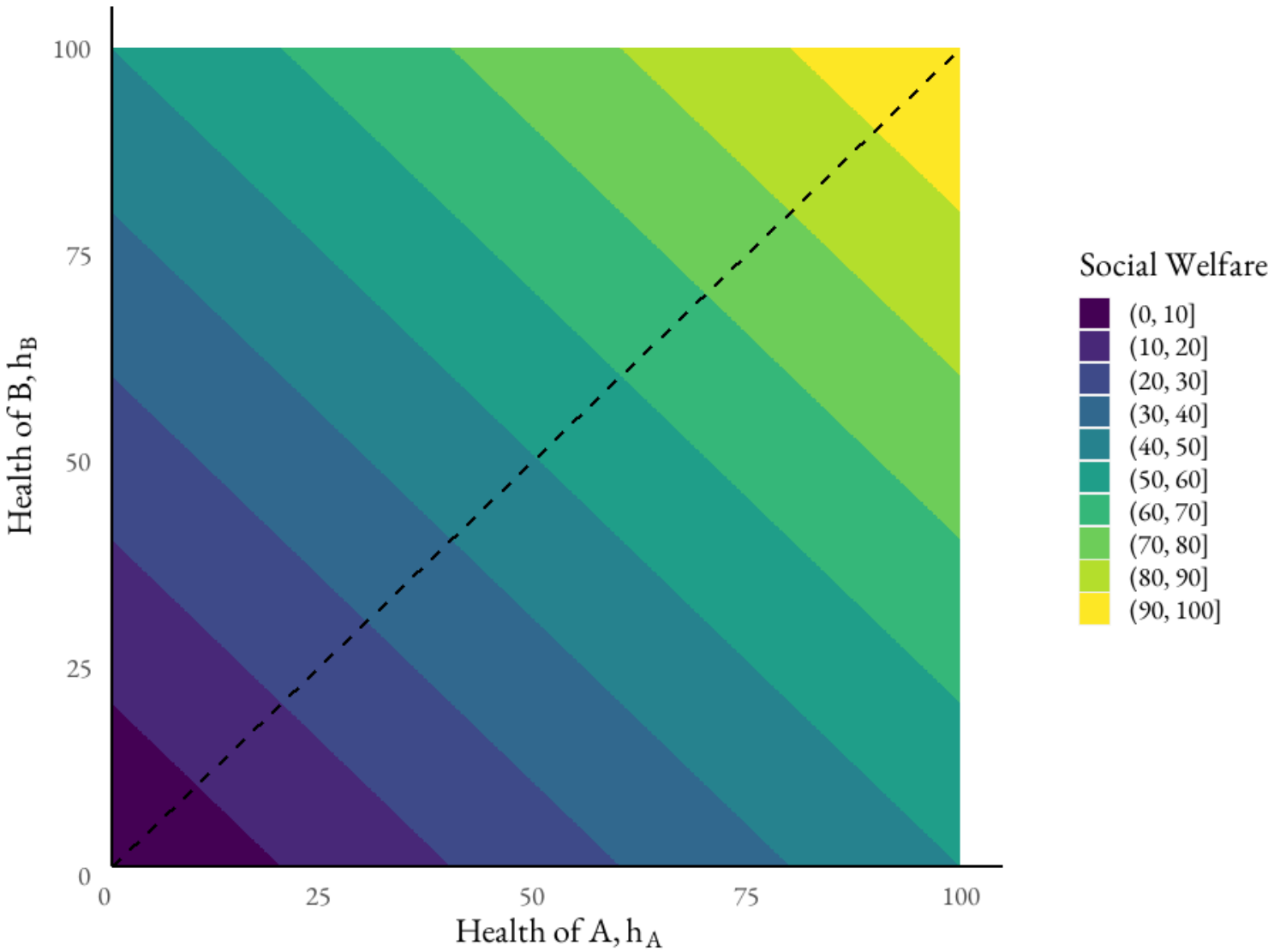


Social Welfare - Atkinson: Health Maximiser

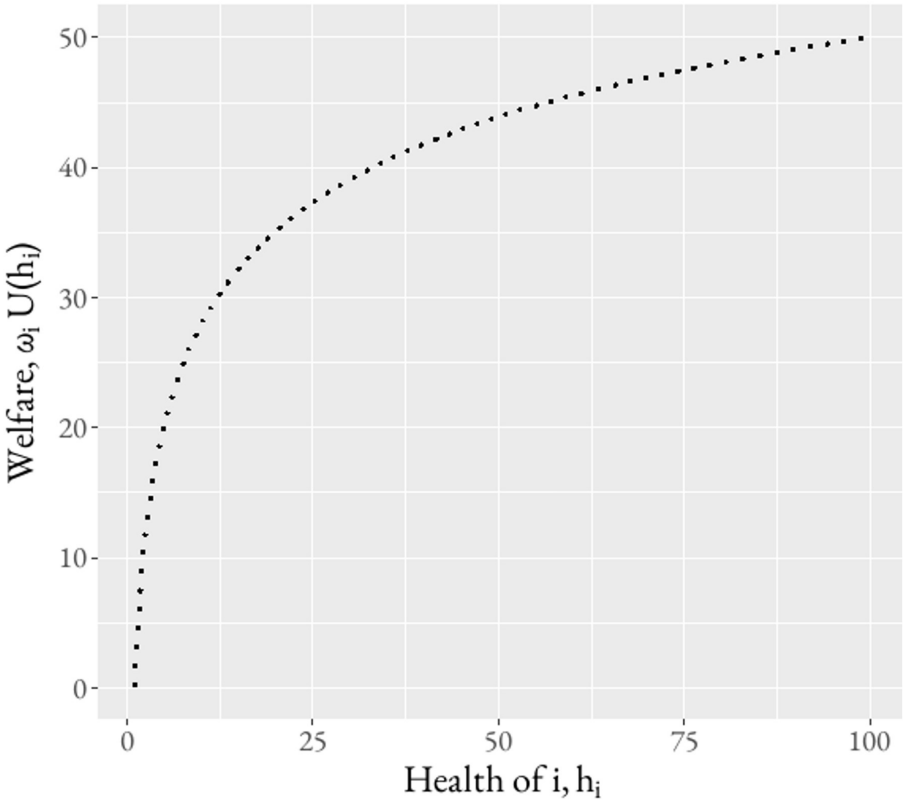


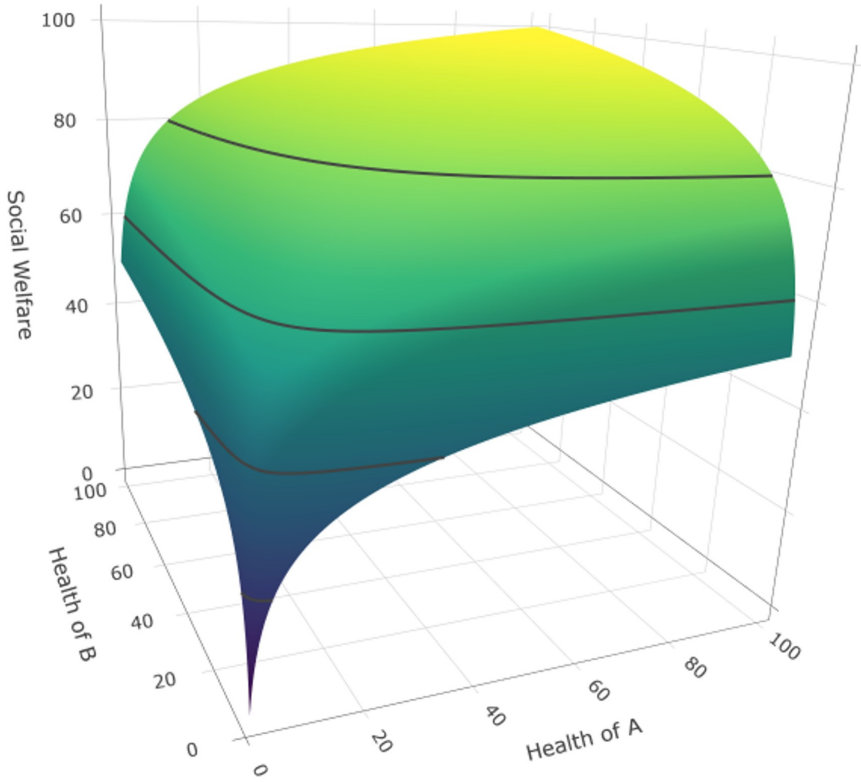


Social Indifference Curves - Health Maximiser

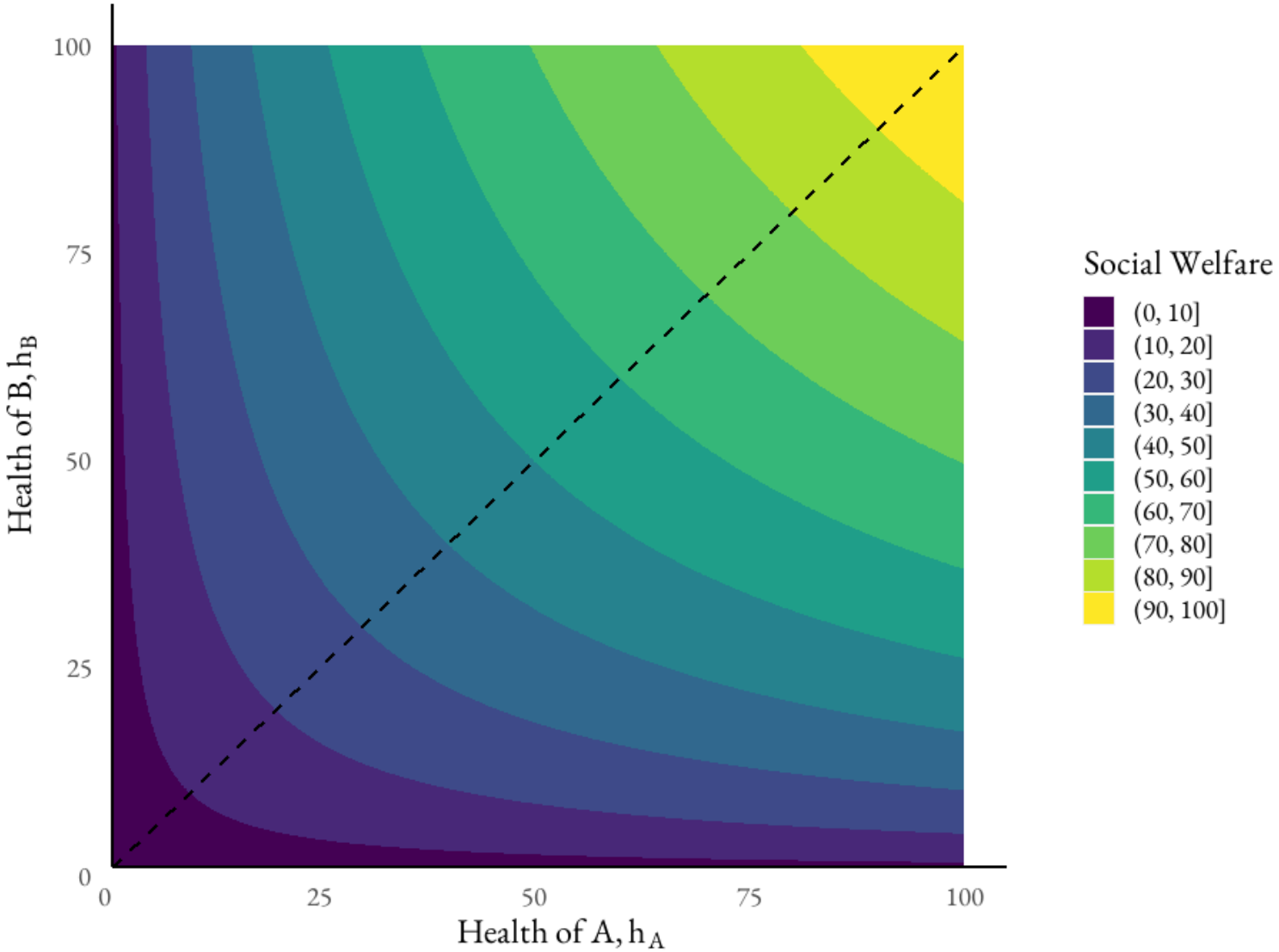


Social Welfare - Atkinson: Prioritarian





Social Indifference Curves - Prioritarian



Alternative Social Indifference Curves

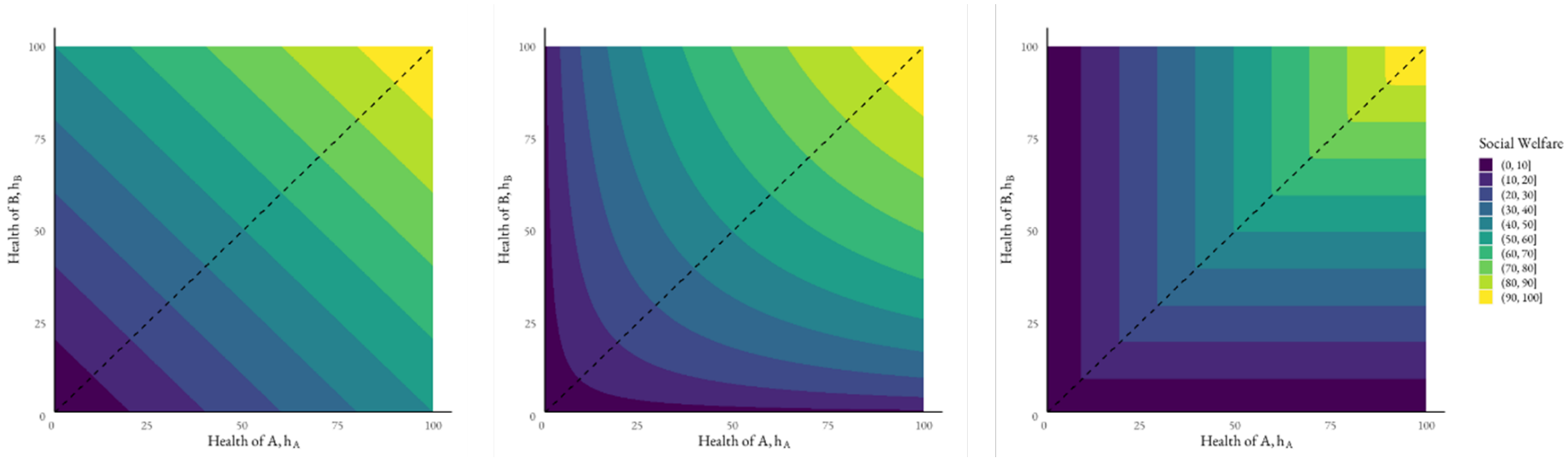
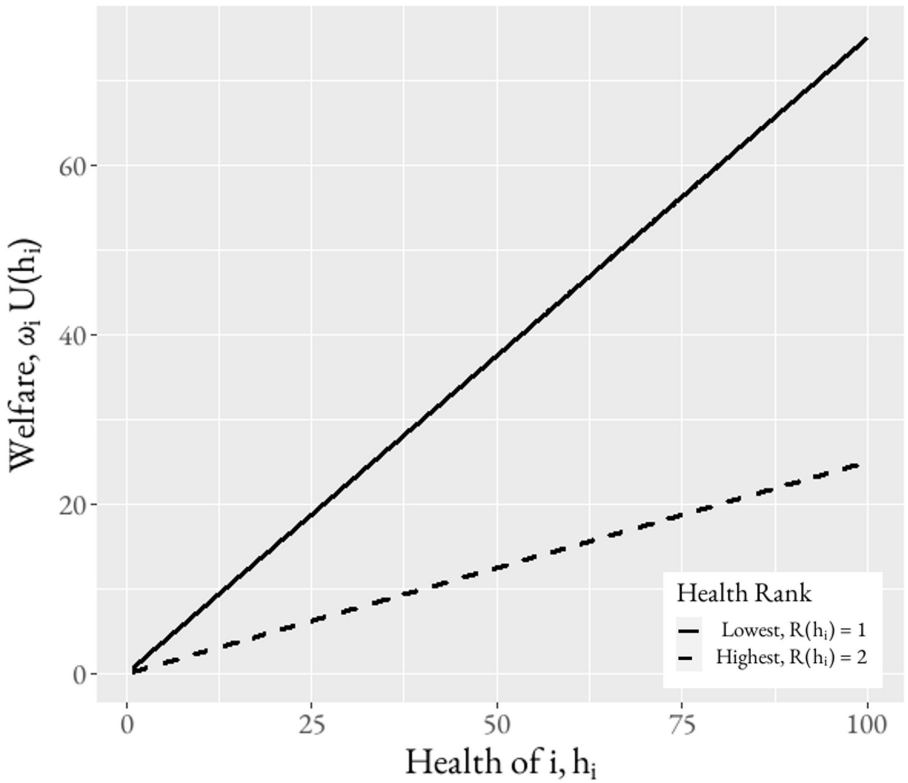
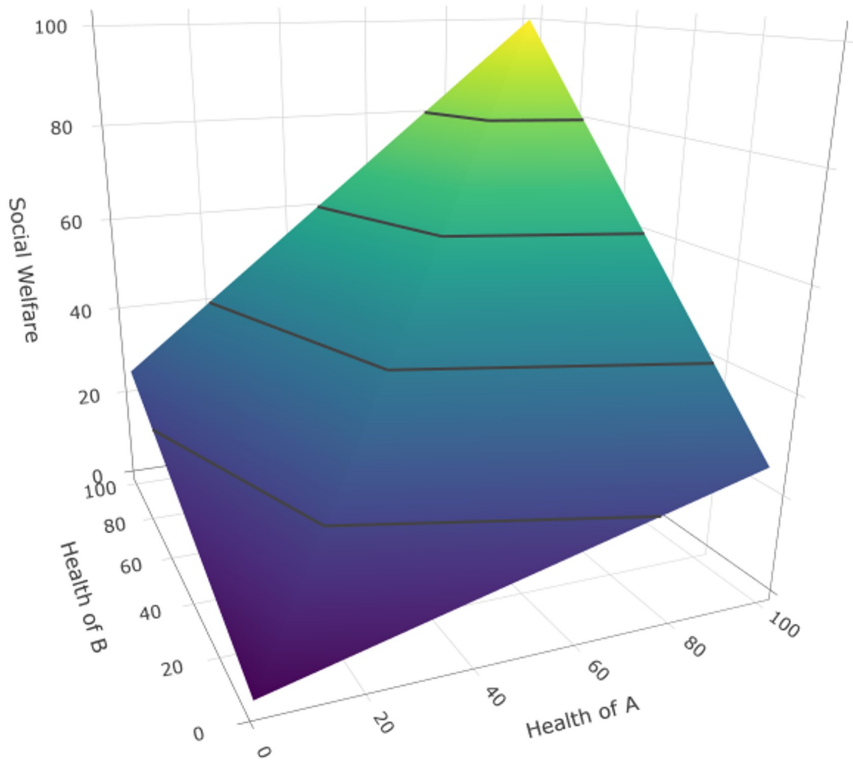


Table: Alternative Social Welfare Functions

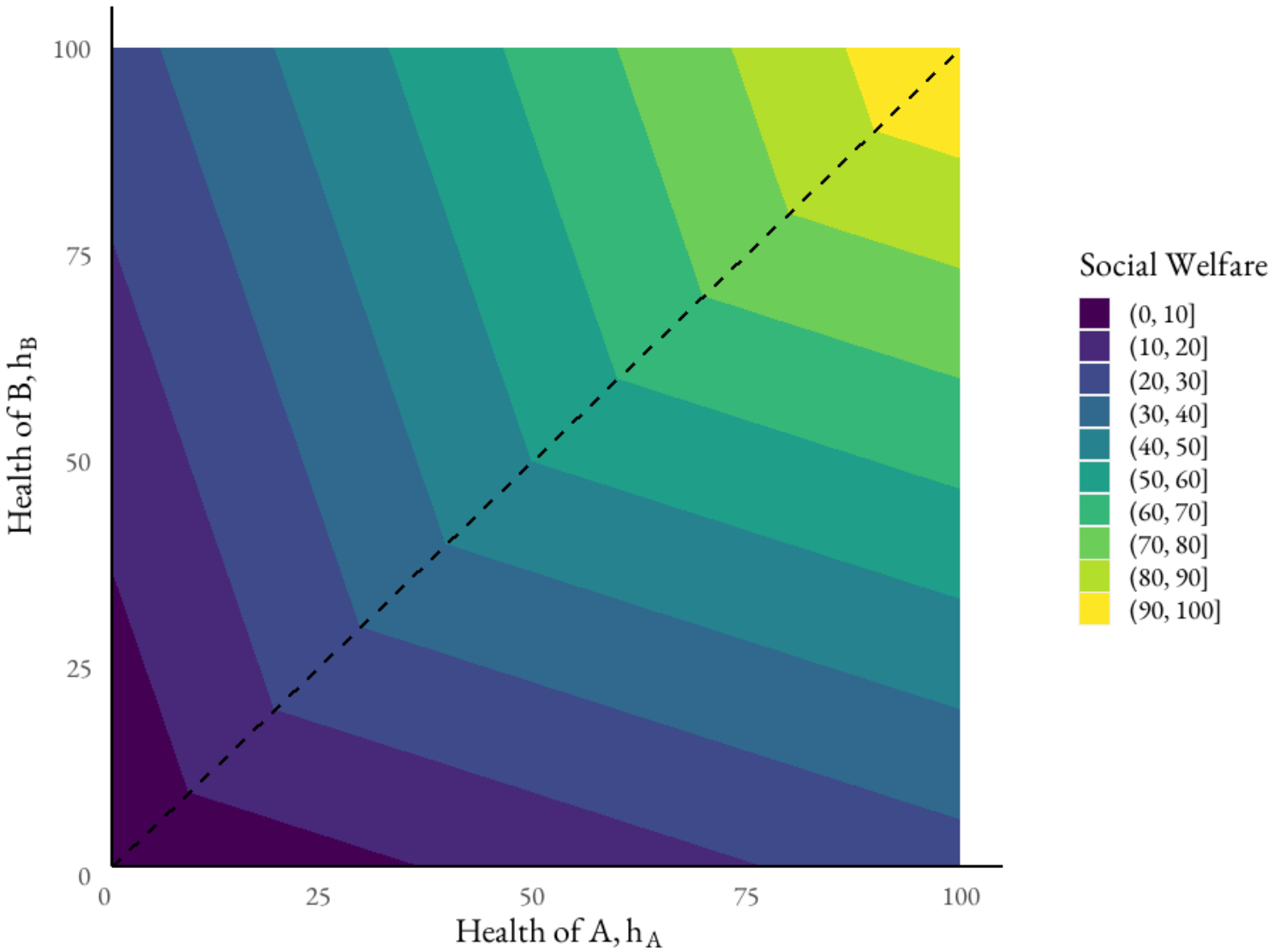
Function	**Weights,** ω_i	**Social Utility,** $U(h_i)$
Atkinson	$\frac{1}{N}$	$\frac{h_i^{1-\varepsilon}-1}{1-\varepsilon}$
Kolm	$\frac{1}{N}$	$-\exp(-\alpha h_i)$
Rank Dependent	$\left(\frac{N-r(h_i)+1}{N}\right)^{\gamma} - \left(\frac{N-r(h_i)}{N}\right)^{\gamma}$	h_i
Socioeconomic-Rank Dependent	$\left(\frac{N-r(x_i)+1}{N}\right)^{\beta} - \left(\frac{N-r(x_i)}{N}\right)^{\beta}$	h_i

Social Welfare - Rank-Dependent

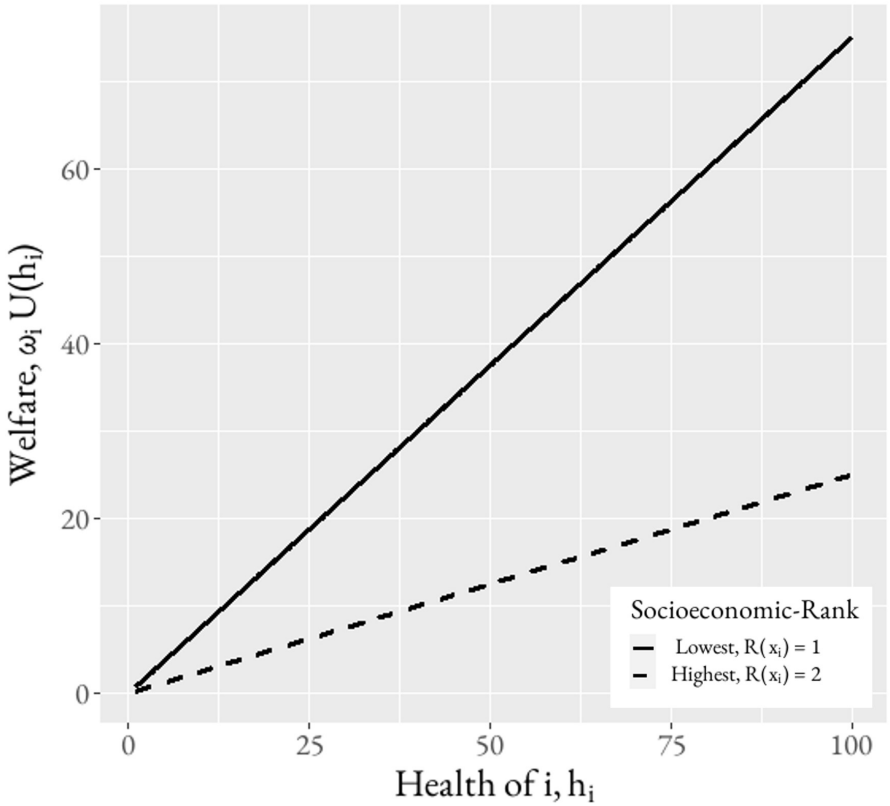


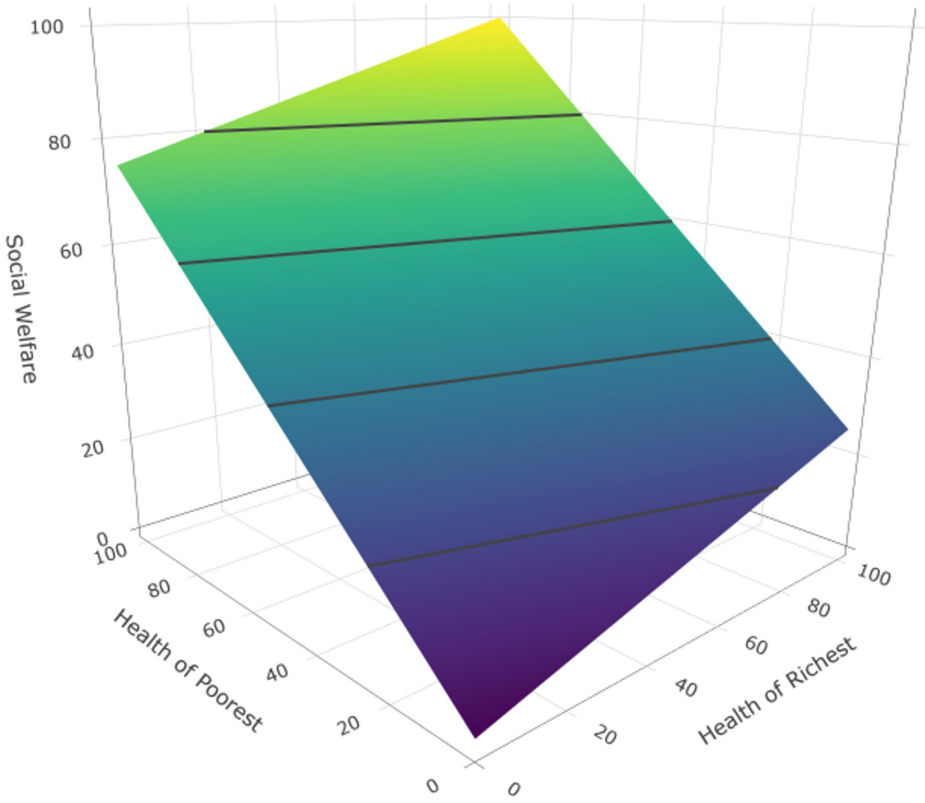


Social Indifference Curves - Rank-Dependent

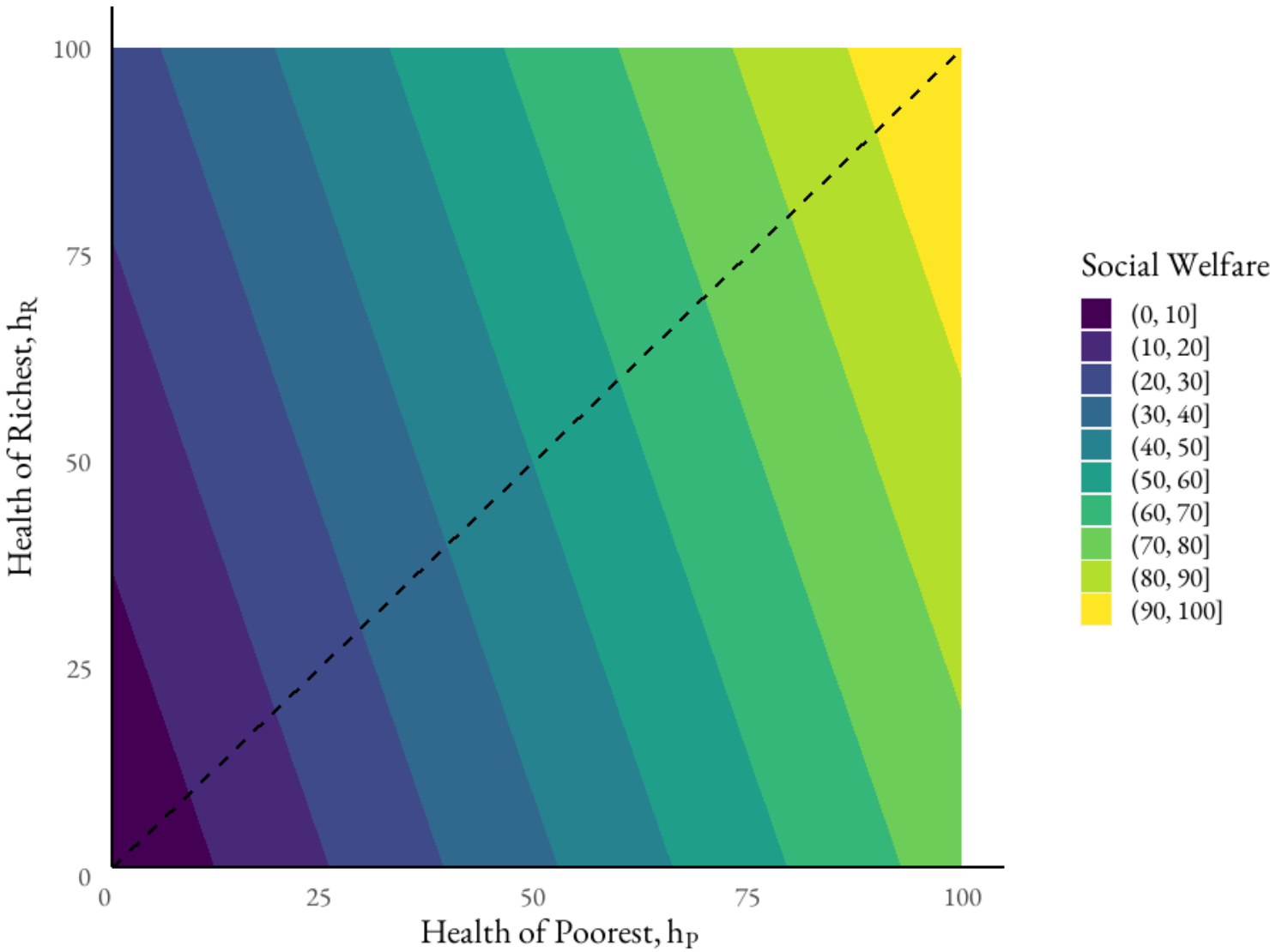


Social Welfare - Rank-SES Dependent

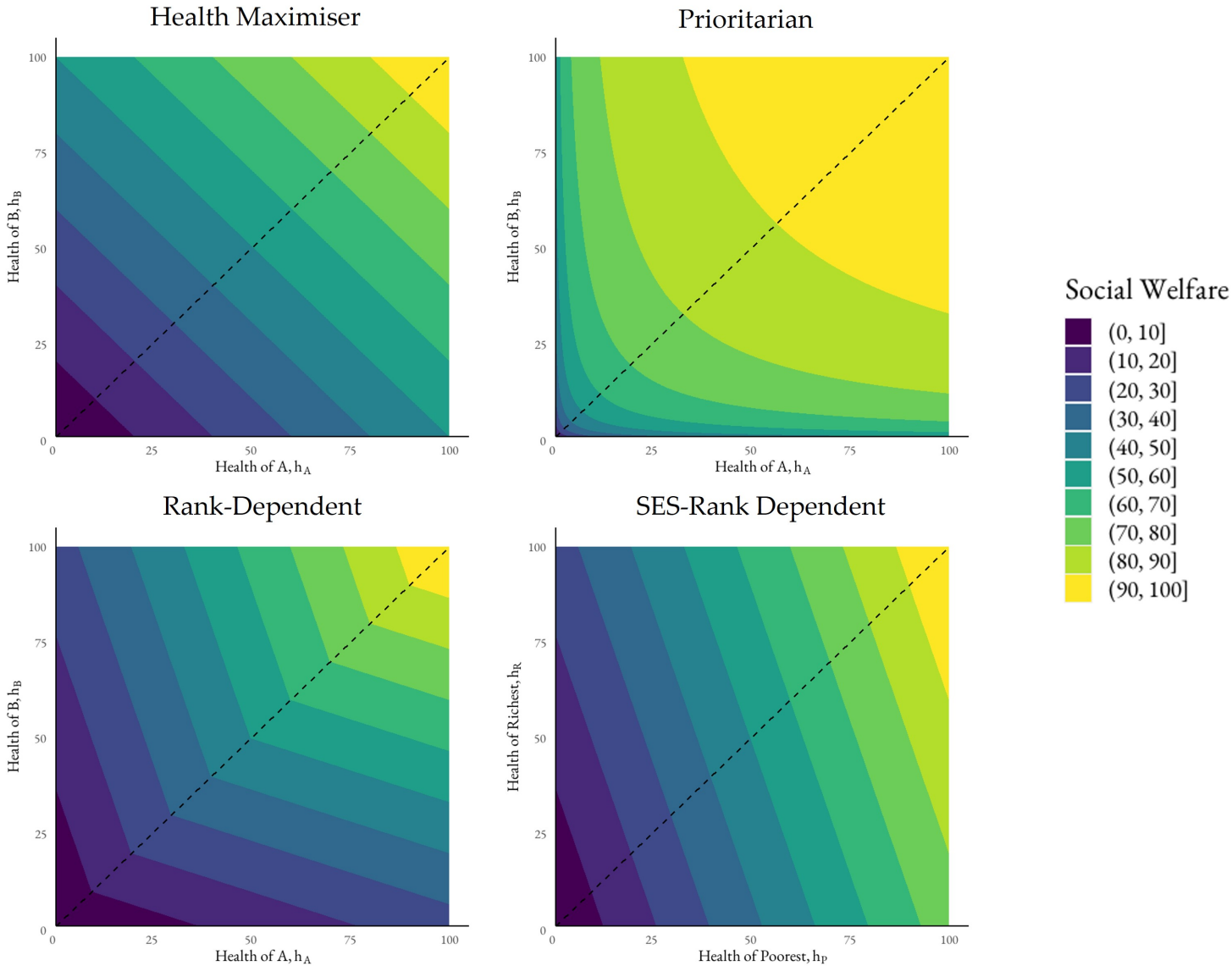




Social Indifference Curves - SES Rank-Dependent



Social Indifference Curves



Optimal Policy Choice

Two questions:

- Which policy is preferred?
- Whose health should be improved?

Optimal Policy Choice

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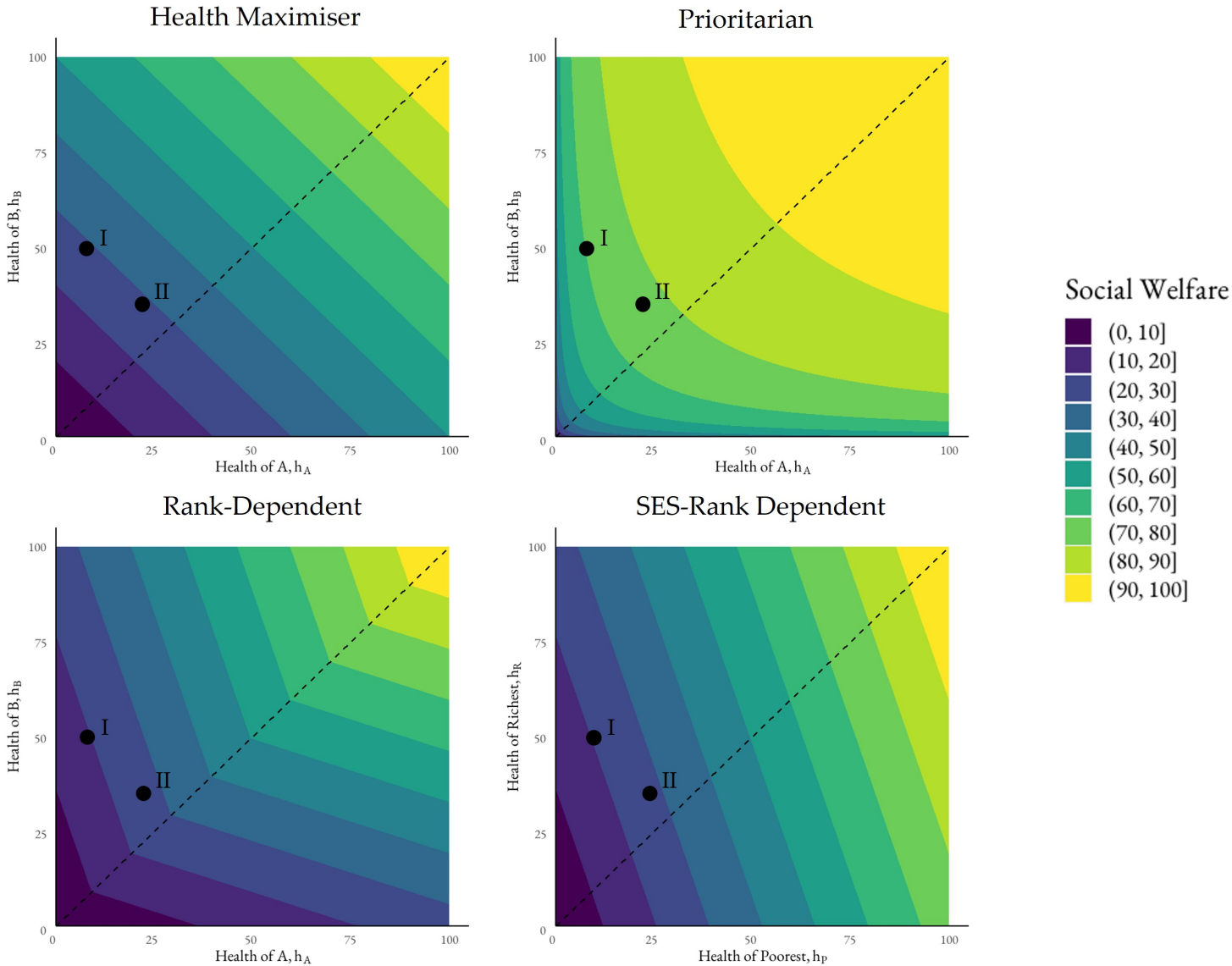
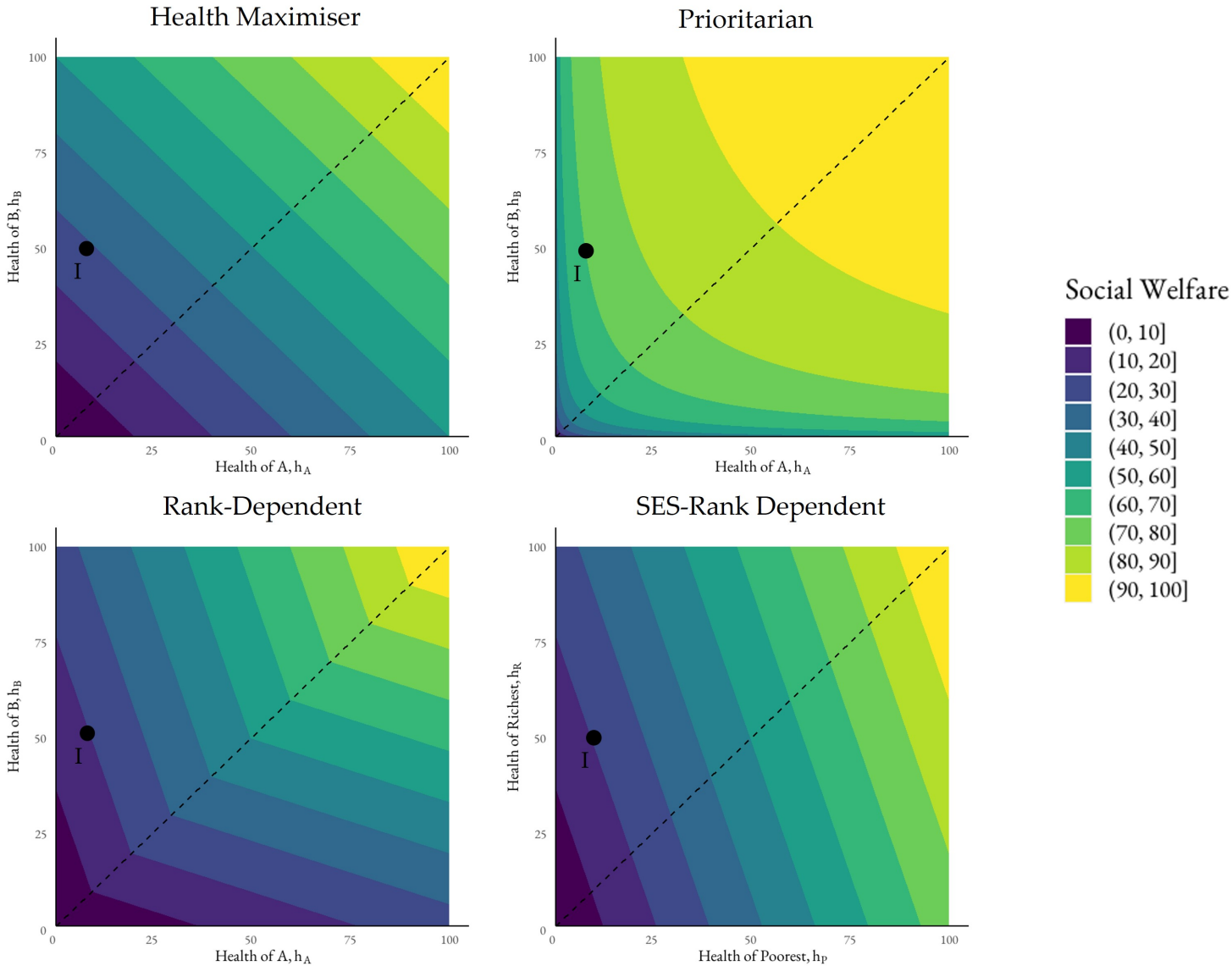
- Which policy is preferred?
- Whose health should be improved?

Answer: Maximise social welfare.

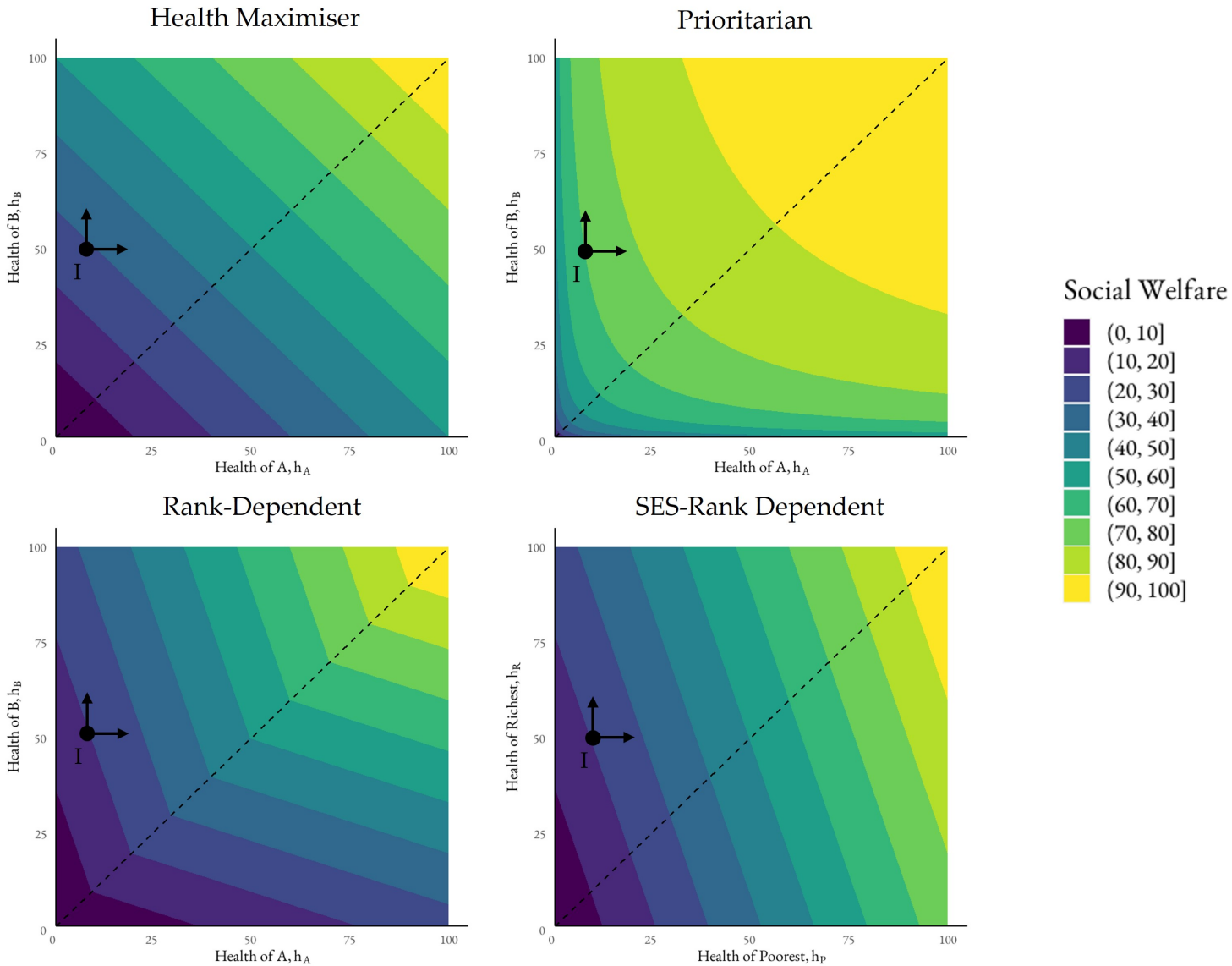
Preferred Policy?



Whose health to improve?



Whose health to improve?



Implied Equity Weight

Implied Equity Weight: $IEW_{ij} = \frac{\partial SW}{\partial h_i} / \frac{\partial SW}{\partial h_j}$

- Relative improvement in social welfare from a marginal improvement in health to i compared to a marginal improvement in health j
- Improve health of i if $IEW_{ij} > 1$

Summary

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- Policy Choice
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- Alternatives
- Policy Choice II